Calculation of variance of number of detected photons in undulator radiation using quantum optics

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1 Main assumptions and conventions

$$K \ll 1 \tag{1}$$
$$N_u \gg 1 \tag{2}$$
$$\overline{n} \gg 1 \tag{3}$$

where K is the undulator's strength parameter, N_u is the number of undulator periods, $\overline{n}$ is the mean number of photons detected per one pass.

Throughout the paper, equations are written in Gaussian units. However, I also omit c everywhere, as if $c = 1$. It can always be restored if needed.

2 Some classical results for undulator radiation

In general, radiation field produced by any classical current far from the source can be found (see [1]) by the following equation

$$\boldsymbol{E}_{\mathrm{tot}}(\boldsymbol{r},\omega) = i\omega\frac{e^{i\omega r}}{r}\sum_{s=1,2}\boldsymbol{e}^{(s)}\otimes\boldsymbol{e}^{(s)}\int d^3\boldsymbol{r}' e^{-i\boldsymbol{k}\cdot\boldsymbol{r}'}\boldsymbol{j}_{\mathrm{tot}}(\boldsymbol{r}',\omega), \tag{4}$$

where $\boldsymbol{k} = \omega\boldsymbol{r}/r$, $\boldsymbol{r}$ is the position of observation point with respect to some reference point in the source of radiation, $\boldsymbol{e}^{(s)}$ are polarization vectors, $\otimes$ denotes tensor product. In our case, the source current is formed by a bunch of electrons. Hence, it is convenient to represent the total field as a sum of contributions from different electrons:

$$\boldsymbol{E}_{\mathrm{tot}}(\boldsymbol{r},\omega) = \sum_j \boldsymbol{E}_j(\boldsymbol{r},\omega). \tag{5}$$

with $j = 1..N_e$, where N_e is the number of electrons in the bunch. An equation for each $E_j(\boldsymbol{r},\omega)$ can easily be found by using the following representation of current produced by jth electron:

$$\boldsymbol{j}^{(j)}(\boldsymbol{r},t) = e\boldsymbol{\beta}^{(j)}(t)\delta^{(3)}\left(\boldsymbol{r}-\boldsymbol{r}^{(j)}(t)\right). \tag{6}$$

The oscillating part of electron's x-component (perpendicular to magnetic field) is

$$x^{(j)}(z^{(j)}) = \frac{K}{\gamma k_u}\sin\left(k_u z^{(j)}\right) \tag{7}$$

From now on, let us focus solely on x-polarization (subscript x will be omitted). Using Eqs. (6) and (7) in Eq. (4) results in

$$E_j(\boldsymbol{r},\omega) = \frac{e}{4\pi}\frac{e^{i\omega r}}{r}\frac{K\omega L_u}{\gamma}(1-\frac{\theta_x^2\omega}{k_u})\,\mathrm{sinc}\left[\pi N_u\left(\frac{\omega}{\omega_0}(1+\gamma^2\theta^2)-1\right)\right]e^{i\omega t_j - i\omega\theta_x x_j - i\omega\theta_y y_j} \equiv \frac{\sqrt{\hbar\omega}e^{i\omega r}}{r}\mathcal{E}(\boldsymbol{n},\omega)e^{-i\omega\boldsymbol{n}\cdot\boldsymbol{r}_j}, \tag{8}$$

where $\omega_0 = 2\gamma^2 k_u$, t_j is the moment of time when jth electron enters the undulator, x_j and y_j are its transverse coordinates at that moment, $\boldsymbol{r}_j \equiv (x_j, y_j, -t_j)$, $\boldsymbol{n} = (\theta_x, \theta_y, 1)$.

3 Calculation of variance of number of detected photons in quantum optics

3.1 Distribution of number of detected photons at a given and fixed $\{\boldsymbol{r}_j\}$

In this subsection we consider one pass of a bunch of electrons (with fixed $\{\boldsymbol{r}_j\}$) through an undulator. The goal is to find the distribution of number of detected photons in this case. In quantum optics, we cannot speak about individual electrons emitting photons, instead there is a certain quantum state of electromagnetic field produced by all the electrons together, and we calculate correlation functions (see [2–4]) of 1st, 2nd,.. orders to find probabilities to detect 1,2,.. photons, respectively. For example, correlation function of the first order is given by

$$G^{(1)}(\boldsymbol{r},t) = \mathrm{Tr}\left[\hat{\rho}\hat{E}^{(-)}(\boldsymbol{r},t)\,\hat{E}^{(+)}(\boldsymbol{r},t)\right], \tag{9}$$

and the probability to detect one photon can be calculated in the following way

$$P(1) \sim \int_{-\infty}^{\infty} dt \int_{\text{detector}} d^2\boldsymbol{r}\, G^{(1)}(\boldsymbol{r},t). \tag{10}$$

Glauber has shown (see [3]) that for any classical current the photon statistics is Poissonian:

$$P(n) = e^{-W}\frac{W^n}{n!}, \tag{11}$$

where the expected value W coincides (for ideal detector) with classical value of mean number of emitted photons n_c:

$$W = n_c = \int_0^{\infty} d\omega \int_{\text{detector}} d^2\boldsymbol{r}\,\frac{1}{\hbar\omega}|E_{\text{tot}}(\boldsymbol{r},\omega)|^2. \tag{12}$$

However, Glauber's result (11) makes sense only when $W \ll 1$. In our case, for a bunch of electrons, if we integrate over entire detector, then $n_c \gg 1$. But we can overcome this obstacle by dividing the entire detector into $N_{\text{det}} \gg 1$ small detectors with equal expected values w, such that w in each of them is much smaller than one. Then, the distribution of number of emitted photons in each of the small detectors will still be Poissonian:

$$p(n) = e^{-\mathrm{w}}\frac{\mathrm{w}^n}{n!}, \tag{13}$$

with

$$\mathrm{w} = \int_0^{\infty} d\omega \int_{\text{small detector i}} d^2\boldsymbol{r}\,\frac{1}{\hbar\omega}|E_{\text{tot}}(\boldsymbol{r},\omega)|^2. \tag{14}$$

According to central limit theorem, the number of photons detected in the entire detector will have a normal distribution with expected value (μ) and variance (σ^2) N_{det} times larger than in the small detectors:

$$\mu = N_{\text{det}}\mathrm{w}, \qquad \sigma^2 = N_{\text{det}}\mathrm{w}, \tag{15}$$

or

$$\mu = \sigma^2 = N_{\text{det}}\mathrm{w} = \sum_i \int_0^{\infty} d\omega \int_{\text{small detector i}} d^2\boldsymbol{r}\,\frac{1}{\hbar\omega}|E_{\text{tot}}(\boldsymbol{r},\omega)|^2 = \int_0^{\infty} d\omega \int_{\text{detector}} d^2\boldsymbol{r}\,\frac{1}{\hbar\omega}|E_{\text{tot}}(\boldsymbol{r},\omega)|^2 = n_c. \tag{16}$$

Thus, the distribution of number of detected photons takes the following form

$$P(n) = \frac{1}{\sqrt{2\pi n_c}} e^{-\frac{1}{2n_c}(n-n_c)^2}, \tag{17}$$

with

$$n_c = \int_0^{\infty} d\omega \int_{\text{detector}} d^2\boldsymbol{r}\,\frac{1}{\hbar\omega}|E_{\text{tot}}(\boldsymbol{r},\omega)|^2, \tag{18}$$

One can use Eqs. (5) and (8) in Eq. (18) to obtain

$$n_c = \int_0^{\infty} d\omega \int_{\text{detector}} d\theta_x d\theta_y |\mathcal{E}(\boldsymbol{n},\omega)|^2 \left(N_e + \sum_{i\neq j} e^{i\omega\boldsymbol{n}(\boldsymbol{r}_i-\boldsymbol{r}_j)}\right). \tag{19}$$

3.2 Taking into account randomness of each $\boldsymbol{r}_j$

In this subsection, we will take into account the fact that each $\boldsymbol{r}_j$ is a random variables, because each turn $\{\boldsymbol{r}_j\}$ change due to synchrotron motion and photon emissions in the rest of the ring. Therefore, now it is important to indicate that n_c is a function of $\{\boldsymbol{r}_j\}$, i.e. $n_c = n_c(\{\boldsymbol{r}_j\})$, the exact dependence is given in Eq. (19). We will assume that t_j,x_j,y_j have independent Gaussian distributions, i.e.

$$\rho(\boldsymbol{r}_j) = \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{1}{2\sigma_z^2}t_j^2} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2\sigma_x^2}x_j^2} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{1}{2\sigma_y^2}y_j^2}, \tag{20}$$

where σ_z is the rms longitudinal bunch size, σ_x and σ_y are transverse rms bunch sizes.

Thus, now, instead of density function (17), we have to work with

$$P(n, \{\boldsymbol{r}_j\}) = \prod_j \rho(\boldsymbol{r}_j) \frac{1}{\sqrt{2\pi n_c(\{\boldsymbol{r}_i\})}} \exp\left[-\frac{1}{2n_c(\{\boldsymbol{r}_i\})}(n - n_c(\{\boldsymbol{r}_i\}))^2\right]. \tag{21}$$

To calculate variance of number of detected photons $\text{var}(n) = \overline{n^2} - \overline{n}^2$, we need to calculate expected value of n ($\overline{n}$) and expected value of n^2 ($\overline{n^2}$). Let us begin with $\overline{n}$:

$$\begin{aligned}\overline{n} &= \int dn \prod_j d^3\boldsymbol{r}_j P(n, \{\boldsymbol{r}_i\}) n = \int \prod_j d^3\boldsymbol{r}_j \rho(\boldsymbol{r}_j) n_c(\{\boldsymbol{r}_i\}) = \\ &= \int_0^\infty d\omega \int_{\text{detector}} d\theta_x d\theta_y |\mathcal{E}(\boldsymbol{n},\omega)|^2 \left(N_e + N_e(N_e-1)e^{-(\sigma_z^2+\theta_x^2\sigma_x^2+\theta_y^2\sigma_y^2)\omega^2}\right) \approx N_e \int_0^\infty d\omega \int_{\text{detector}} d\theta_x d\theta_y |\mathcal{E}(\boldsymbol{n},\omega)|^2.\end{aligned} \tag{22}$$

In the above, first, integration over n is performed, then Eqs. (19) and (20) are used, and integration over $\{\boldsymbol{r}_j\}$ is performed. Finally, the contribution with $N_e(N_e-1)$ was neglected, because $\sigma_z\omega_0 \sim 6\times10^5$ ($\sigma_z \sim 5\,\text{cm}$,$\lambda_0 \sim 500\,\text{nm}$). Analogously, one can calculate $\overline{n^2}$:

$$\overline{n^2} = \int dn \prod_j d^3\boldsymbol{r}_j P(n, \{\boldsymbol{r}_i\}) n^2 = \int \prod_j d^3\boldsymbol{r}_j \rho(\boldsymbol{r}_j)\left(n_c^2(\{\boldsymbol{r}_i\}) + n_c(\{\boldsymbol{r}_i\})\right) = \int \prod_j d^3\boldsymbol{r}_j \rho(\boldsymbol{r}_j) n_c^2(\{\boldsymbol{r}_i\}) + \overline{n}. \tag{23}$$

Using Eq. (19),

$$\begin{aligned}n_c^2(\{\boldsymbol{r}_i\}) = \int_0^\infty d\omega_1 d\omega_2 \int_{\text{detector}} d\theta_{1x}d\theta_{1y}d\theta_{2x}d\theta_{2y}|\mathcal{E}(\boldsymbol{n}_1,\omega_1)|^2|\mathcal{E}(\boldsymbol{n}_2,\omega_2)|^2 \Bigg(N_e^2 + N_e\sum_{i\neq j} e^{i\omega_1\boldsymbol{n}_1(\boldsymbol{r}_i-\boldsymbol{r}_j)} + \\ N_e \sum_{n\neq m} e^{-i\omega_2\boldsymbol{n}_2(\boldsymbol{r}_n-\boldsymbol{r}_m)} + \sum_{\substack{i\neq j, n\neq m\\ i,j\neq n,m}} e^{i\omega_1\boldsymbol{n}_1(\boldsymbol{r}_i-\boldsymbol{r}_j)-i\omega_2\boldsymbol{n}_2(\boldsymbol{r}_n-\boldsymbol{r}_m)} + \sum_{i\neq j} e^{i(\omega_1\boldsymbol{n}_1-\omega_2\boldsymbol{n}_2)(\boldsymbol{r}_i-\boldsymbol{r}_j)}\Bigg).\end{aligned} \tag{24}$$

Again, because $\sigma_z\omega_0 \sim 6\times10^5$, second, third, and fourth terms in Eq. (24) will give negligible contributions to Eq. (23). However, the last term in Eq. (24) will give nonzero contribution (along with the first one, obviously, which will give $\overline{n}^2$). Thus,

$$\overline{n^2} = \overline{n}^2 + \overline{n} + \Delta, \tag{25}$$

where

$$\begin{aligned}\Delta &= \int_0^\infty d\omega_1 d\omega_2 \int_{\text{detector}} d\theta_{1x}d\theta_{1y}d\theta_{2x}d\theta_{2y}|\mathcal{E}(\boldsymbol{n}_1,\omega_1)|^2|\mathcal{E}(\boldsymbol{n}_2,\omega_2)|^2 \int \prod_n d^3\boldsymbol{r}_n \rho(\boldsymbol{r}_n) \sum_{i\neq j} e^{i(\omega_1\boldsymbol{n}_1-\omega_2\boldsymbol{n}_2)(\boldsymbol{r}_i-\boldsymbol{r}_j)} = \\ &= N_e(N_e-1)\int_0^\infty d\omega_1 d\omega_2 \int_{\text{detector}} d\theta_{1x}d\theta_{1y}d\theta_{2x}d\theta_{2y}|\mathcal{E}(\boldsymbol{n}_1,\omega_1)|^2|\mathcal{E}(\boldsymbol{n}_2,\omega_2)|^2 e^{-\sigma_z^2(\omega_1-\omega_2)^2-\sigma_x^2(\omega_1\theta_{1x}-\omega_2\theta_{2x})^2-\sigma_y^2(\omega_1\theta_{1y}-\omega_2\theta_{2y})^2},\end{aligned} \tag{26}$$

where $e^{-\sigma_z^2(\omega_1-\omega_2)^2}$ acts like a delta function, because $\frac{1}{\sigma_z} \ll \frac{\omega_0}{N_u}$. However, the remaining two exponents do not behave akin to delta functions, because $\sigma_\perp \omega_0 \frac{1}{\gamma} \sim 3$ (see Sec. 4). Therefore,

$$\Delta = N_e(N_e-1)\frac{\sqrt{\pi}}{\sigma_z}\int_0^\infty d\omega \int_{\text{detector}} d\theta_{1x}d\theta_{1y}d\theta_{2x}d\theta_{2y}|\mathcal{E}(\boldsymbol{n}_1,\omega)|^2|\mathcal{E}(\boldsymbol{n}_2,\omega)|^2 e^{-\omega^2\sigma_x^2(\theta_{1x}-\theta_{2x})^2-\omega^2\sigma_y^2(\theta_{1y}-\theta_{2y})^2}. \tag{27}$$

The above integral should be estimated numerically in general case. And then the normalized variance can be calculated by

$$\frac{\mathrm{var}(n)}{\overline{n}^2} = \frac{1}{\overline{n}} + \frac{\Delta}{\overline{n}^2}. \tag{28}$$

Before we move on, it is useful for numerical calculations to express $\overline{n}$ and Δ through dimensionless integrals (using Eq. (8))

$$\overline{n} = N_e\alpha\gamma^2 K N_u^2 I_1, \qquad \Delta = N_e(N_e-1)\left(\alpha\gamma^2 K N_u^2\right)^2 \frac{\sqrt{\pi}}{\sigma_z\omega_0} I_2, \tag{29}$$

where

$$I_1 = \int d\tilde{\omega}d\theta_x d\theta_y \tilde{\omega}(1-2\gamma^2\theta_x^2\tilde{\omega})^2 \operatorname{sinc}^2\left[\pi N_u\left(\tilde{\omega}(1+\gamma^2\theta^2)-1\right)\right], \tag{30}$$

$$I_2 = \int d\tilde{\omega}d\theta_{1x}d\theta_{1y}d\theta_{2x}d\theta_{2y}\tilde{\omega}^2(1-2\gamma^2\theta_{1x}^2\tilde{\omega})^2(1-2\gamma^2\theta_{2x}^2\tilde{\omega})^2 \times \operatorname{sinc}^2\left[\pi N_u\left(\tilde{\omega}(1+\gamma^2\theta_1^2)-1\right)\right]\operatorname{sinc}^2\left[\pi N_u\left(\tilde{\omega}(1+\gamma^2\theta_2^2)-1\right)\right] e^{-(\sigma_x\omega_0)^2\tilde{\omega}^2(\theta_{1x}-\theta_{2x})^2-(\sigma_y\omega_0)^2\tilde{\omega}^2(\theta_{1y}-\theta_{2y})^2}, \tag{31}$$

with $\tilde{\omega} = \omega/\omega_0$. Now it is clear that $\Delta \propto \overline{n}^2$ ($N_e - 1 \approx N_e$, because $N_e \gg 1$). Hence, $\Delta/\overline{n}^2$ does not depend on $\overline{n}$, and at some point (as $\overline{n}$ grows) $\Delta/\overline{n}^2$ will exceed $1/\overline{n}$ in Eq. (28):

$$\frac{\mathrm{var}(n)}{\overline{n}^2} = \frac{1}{\overline{n}} + \frac{\sqrt{\pi}}{\sigma_z\omega_0}\frac{I_2}{I_1^2} \tag{32}$$

3.3 Taking into account photodiode detection mechanism

A photodiode has certain probability distribution $P(n_e)$ to produce n_e electrons after detection of one photon. Let us denote expected value and variance of this distribution by μ_e and σ_e^2, respectively. Now, one can repeat all the above derivations, but for number of produced electrons in the photodiode (since this is what we measure on experiment), instead of number of detected photons. Calculations will be very similar with the exception that the density function (21) will take the following form

$$P\left(n_e, n, \{\boldsymbol{r}_j\}\right) = \frac{1}{\sqrt{2\pi n}\sigma_e}\exp\left[-\frac{1}{2n\sigma_e^2}\left(n_e - n\mu_e\right)^2\right] \times P\left(n, \{\boldsymbol{r}_j\}\right). \tag{33}$$

The reason why (21) is modified in this way is that when $n \gg 1$ photons are detected, the probability distribution for produced number of electrons n_e is normal with expected value and variance equal to $n\mu_e$ and $n\sigma_e^2$, respectively, due to the central limit theorem.

One can easily deduce from Eq. (33), that

$$\overline{n}_e = \int dn_e dn \prod_j d^3\boldsymbol{r}_j P\left(n_e, n, \{\boldsymbol{r}_i\}\right) n_e = \mu_e \overline{n}, \tag{34}$$

$$\overline{n_e^2} = \int dn_e dn \prod_j d^3\boldsymbol{r}_j P\left(n_e, n, \{\boldsymbol{r}_i\}\right) n_e^2 = \mu_e^2\overline{n^2} + \sigma_e^2\overline{n}, \tag{35}$$

where $\overline{n}$ and $\overline{n^2}$ have already been calculated above. Hence, it is straightforward to obtain a normalized variance of number of produced electrons (analogous to Eq. (32))

$$\frac{\mathrm{var}(n_e)}{\overline{n}_e^2} = \frac{\mu_e + \sigma_e^2/\mu_e}{\overline{n}_e} + \frac{\sqrt{\pi}}{\sigma_z\omega_0}\frac{I_2}{I_1^2}. \tag{36}$$

It is also important to estimate the effect of noise in the photodiode. Let us denote the number of electrons produced by the electron beam per one turn by n_{eb}, and the number of electrons due to noise in the photodiode by n_{en}, where both n_{eb} and n_{en} are independent random variables. Total number of produced electrons is $n_e = n_{eb}+n_{en}$. Then, $\mathrm{var}(n_e) = \mathrm{var}\,(n_{eb}) + \mathrm{var}\,(n_{en})$. Hence, Eq. (36) changes to

$$\frac{\mathrm{var}(n_e)}{\overline{n}_e^2} = \frac{\mu_e + \sigma_e^2/\mu_e}{\overline{n}_e} + \frac{\sqrt{\pi}}{\sigma_z\omega_0}\frac{I_2}{I_1^2} + \frac{\mathrm{var}\,(n_{en})}{\overline{n}_e^2}, \tag{37}$$

and the noise contribution falls as negative second power of $\overline{n}_e$, and can be neglected for decent photodiodes and $\overline{n}_e \gg 1$. For example, for [5], $n_{en} \sim 1-10$ (see Appendix A), n_e can be $\sim 10^6$ and greater (see Sec. 4). Therefore, $\mathrm{var}\,(n_{en})/\overline{n}_e^2 < 10^{-11}$, and it can definitely be neglected (see Sec. 4). Thus, we can omit the noise term at this point and never return to it.

4 Numerical estimations

Let us assume the following values of parameters for IOTA:

- $N_e = 10^9$
- $\gamma = 235$
- $K = 0.1$
- $N_u = 10$
- $\lambda_u = 5.5\,\mathrm{cm}$
- $\lambda_0 = \lambda_u/(2\gamma^2) = 500\,\mathrm{nm}$
- $\sigma_z = 5\,\mathrm{cm}$
- $\sigma_x = \sigma_y = 100\,\mathrm{\mu m}$
- $\Delta\theta_x = \Delta\theta_y = 4/\gamma$ (square detector)
- $\mu_e = 0.87$, (see [5])
- $\sigma_e^2 = \mu_e = 0.87$

In this case, in Eq. (36),

$$\frac{\mu_e + \sigma_e^2/\mu_e}{\overline{n}_e} = 2.5\times10^{-6}, \qquad \frac{\sqrt{\pi}}{\sigma_z\omega_0}\frac{I_2}{I_1^2} = 0.7\times10^{-6}. \tag{38}$$

Thus, the two contributions are comparable, and this can be observed on experiment. We will probably be able to see the second term become even much bigger than the first one at bigger K, but in this simple theoretical model we cannot use bigger K, because to use Eq. (8), we assumed that $K \ll 1$. In the paper [6], they had a wiggler with $K > 1$.

5 The limit of a pinhole detector

In the limiting case, when

$$\theta_D \ll 1/\left(\sqrt{N_u}\gamma\right) \sim 10^{-3}, \tag{39}$$

$$\theta_D \ll \lambda_0/\left(2\pi\sigma_\perp\right) \sim 10^{-3}, \tag{40}$$

the integrals (30) and (31) can be calculated analytically, and the variance takes the simple form

$$\frac{\mathrm{var}(n_e)}{\overline{n}_e^2} = \frac{\mu_e + \sigma_e^2/\mu_e}{\overline{n}_e} + \frac{1}{3\sqrt{\pi}}\frac{T_p}{\sigma_z}, \tag{41}$$

where $T_p = N_u \lambda_0$ is the length of the pulse of electric field produced by each electron. It can be shown by taking a Fourier transform of Eq. (8) on z-axis ($r = z$):

$$E_j(\boldsymbol{r}, t) = \int E_j(\boldsymbol{r}, w) e^{-i\omega} d\omega = \frac{e}{2\pi r} \frac{K \lambda_u \omega_0^2}{\gamma \pi} \operatorname{rect}\left[\frac{\omega_0 \left(r - (t - t_j)\right)}{2N_u}\right] \cos\left[\omega_0 \left(r - (t - t_j)\right)\right] \tag{42}$$

where

$$\operatorname{rect}[x] = \begin{cases} 1, & \text{if } |x| \leq 1, \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

However, we cannot use a pinhole detector in IOTA, because then the second term in Eq. (41) is much smaller than the first one. Indeed, for $\theta_D = 10^{-4}$ and other parameters' values from Sec. 4

$$\frac{\mu_e + \sigma_e^2/\mu_e}{\overline{n}_e} = 3.0 \times 10^{-3}, \qquad \frac{1}{3\sqrt{\pi}} \frac{T_p}{\sigma_z} = 1.9 \times 10^{-5}. \tag{44}$$

6 Experimental setup

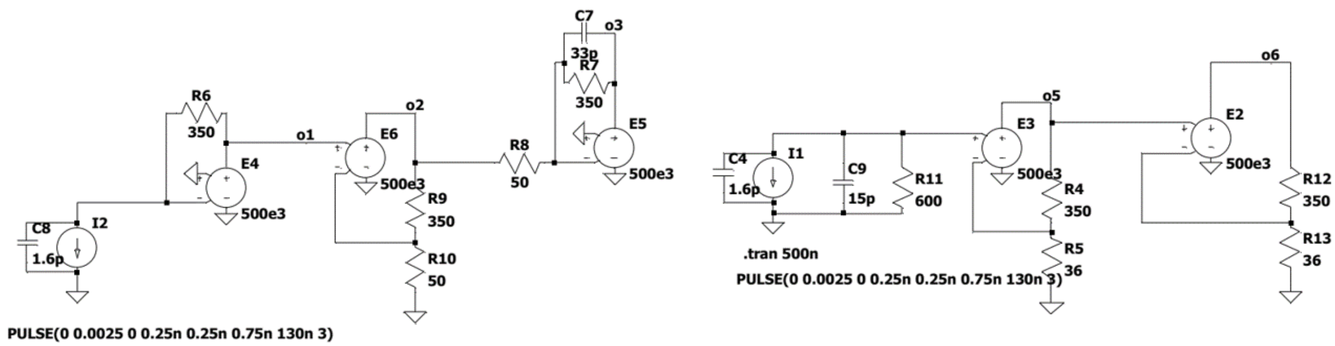

Figure 1: Two possible schemes of an integrator for the experiment (the right scheme will be used).

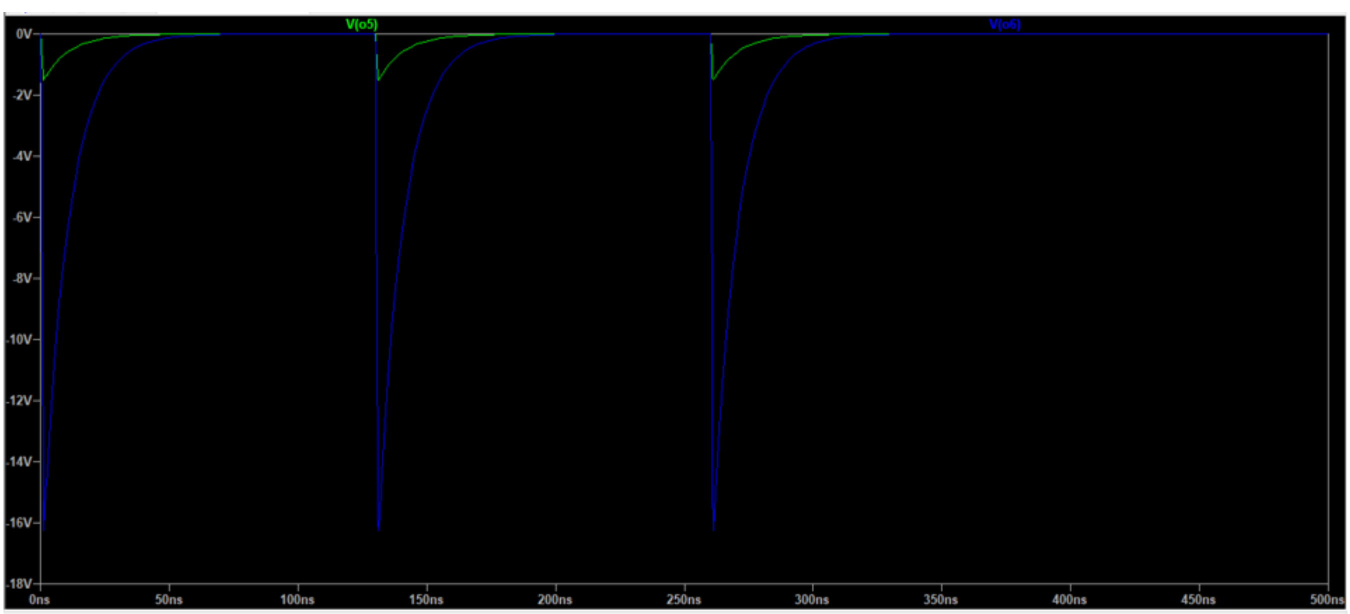

Figure 2: Voltage after first opamp (green) and votage after second opamp (blue).

6.1 Johnson–Nyquist noise

Voltage variance due to Johnson–Nyquist noise can be found by

$$\mathrm{var}\,(V_{JN}) = 4k_B T_0 R\sqrt{\Delta f} = 38\,\mu\mathrm{V}, \tag{45}$$

where $\Delta f = 75\,\mathrm{MHz}$ (ten times the IOTA revolution frequency), $R = 600\,\Omega$, $T_0 = 300\,\mathrm{K}$.

This converts to the following detected electron number variance per turn

$$\mathrm{var}\,(n_{eJN}) = \frac{\mathrm{var}\,(V_{JN})T}{Re} \sim 10^4, \tag{46}$$

which means that the Johnson–Nyquist noise contribution to normalized variance of electron number is negligible:

$$\frac{\mathrm{var}\,(n_{eJN})}{\overline{n}_e^2} = \frac{10^4}{\overline{n}_e^2} = 10^{-8} \ll \frac{1}{\overline{n}_e} = 10^{-6}. \tag{47}$$

A Photodiode noise

At 410 nm (see [5]),

$$n_{en} = \frac{1.9\times 10^{-15}\,\mathrm{W/Hz^{1/2}} \times \sqrt{75\,\mathrm{MHz}} \times 0.3\,\mathrm{A\,W^{-1}} \times 132\,\mathrm{ns}}{1.6\times 10^{-19}\,\mathrm{C}} \approx 4. \tag{48}$$

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